Mechanical Problem Set #4 - Compliant Grid for Stage Crashes

Executive Summary:

We designed a rectilinear grid of compliant members to maximize energy absorption during a crash event for an ASML stage. To achieve a nonlinear stiffness which would maximize energy, we converged on a diamond shape connecting each node. We expected that as the diamond would compress, it would exhibit two distinct regimes–one associated with axial loading (higher initial stiffness), and the other with bending (lower final stiffness)--as the angle with respect to the horizontal decreased (see Appendix). We then derived a stiffness matrix for one diamond, and subsequently produced surface plots for a global grid stiffness, force, and energy to determine optimal geometric parameters. Our final, scaled down prototype is composed of TPU with L = 25.4 mm, h = 2 mm, b = 5 mm, and θ_{start} = 70 degrees. capable of absorbing 1.52 J.

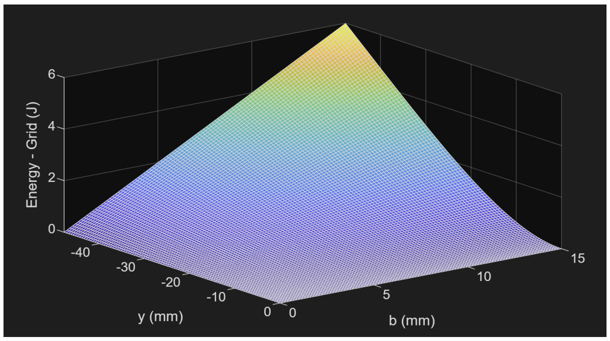
Math Model:

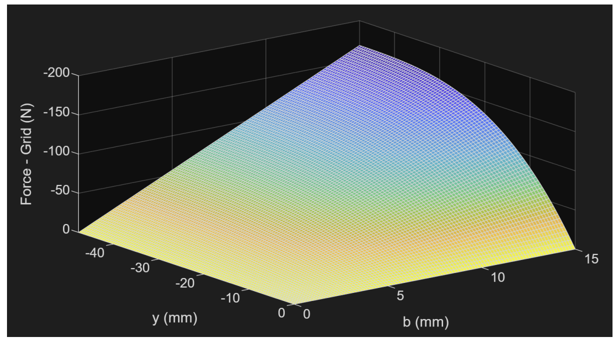
We first focused on determining a 12x12 global stiffness matrix for one diamond (see Appendix), which contains 4 nodes with 3 separate degrees of freedom (x, y, θ). The global matrix was obtained by superimposing 4, 6x6 local stiffness matrices for each bar in the diamond. The local stiffness matrices, **K**, were determined by applying a transformation matrix to a beam "frame" element stiffness matrix. This operation is illustrated in the appendix, where θ is the angle that each respective bar makes with the horizontal.

To find the net y-stiffness of a diamond, $K_{diamond}$, we first computed F2,y by evaluating **F = K*u** and taking the fifth component of **F** to calculate $dF2, y/dy2$. To obtain the global stiffness of the entire grid, we modeled each diamond as a spring with stiffness $K_{diamond}$; then, by applying arithmetic rules for springs in series and parallel, we determined that K_{grid} = $j/i * K_{diamond}$, where *j* is the number of columns present in the grid, and *i* is the number of rows.

With an expression for the grid stiffness, we proceeded with determining a maximum deflection that could be obtained without overstressing the material. Modeling the individual beams in each diamond as undergoing fixed-guided bending, the stress in an individual beam was determined to be $\sigma_{beam} = 1/L^2 * 6Ec * \delta cos(\theta_{start})$, where $\delta = Y_{max}/(2 * i)$. To simplify the optimization process, L and c were fixed at 25.4 and 1.5 mm, respectively. With this method, PLA produced Y_{max} values that were too low to demonstrate a nonlinear stiffness over the allowable range of deflection. Switching to TPU remedied this, allowing much larger Y_{max} values to be achieved before overstressing the material; therefore, our calculation for maximum deflection became simplified to $Y_{max} = 2 * L * sin(\theta_{start})$, i.e. the starting vertical length of the diamond.

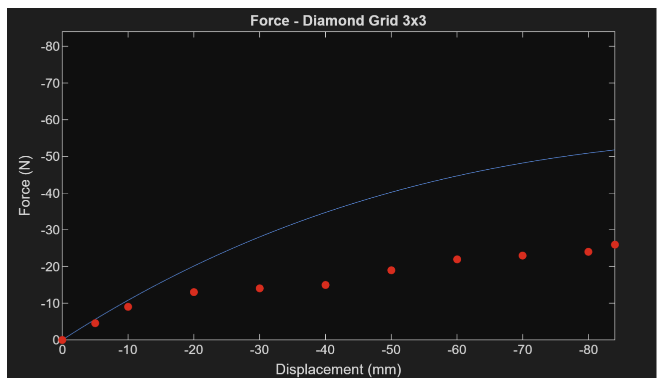
With a Y_{max} at hand, we then produced surface plots for force, stiffness, and energy as a function of a global Y (see Appendix for global coordinate frame), as well as other parameters which we wanted to optimize, such as starting angle (θ_{start}), depth (b). Before producing surface plots, we re-expressed $K_{diamond}$ in terms of Y, not θ. This was achieved by substituting in $\theta = arcsin(sin(\theta_{start} + (Y/i)/(2 * L))$.





Verification Method and Results:

Using our testing setup (see Appendix), we measured the force required for our crash pad to reach different displacement intervals. Plotted against the Force vs. Displacement graph from our model (shown in blue) we found that the observed behavior tracks our model well for small displacements (approximately < 10mm). However, for larger displacements, the observed forces begin to level off at a faster rate compared to our model. One reason this may occur is because the beam bending equations we employed become less accurate for larger displacements. Our observations also show that the stiffness achieves the non-linear effect we desired, initially large and decreasing as we reach a maximum compression.

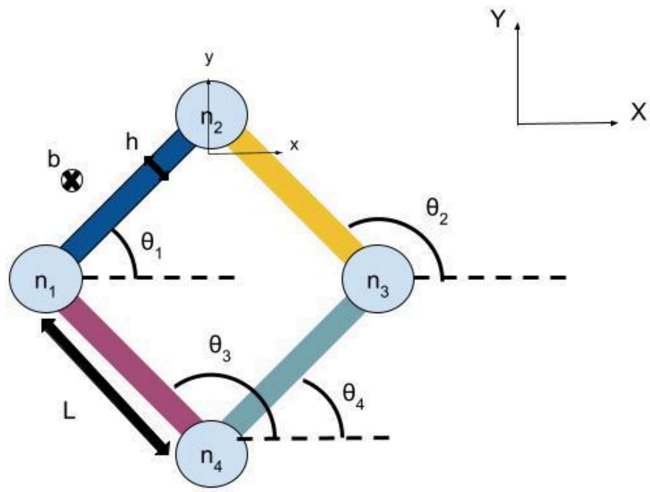


Discussion/Learnings:

During the early phase of our analysis, we attempted to pressure test the stiffness matrix that we derived for a single diamond by applying relevant boundary conditions (fixing node 4, downward force on node 2, etc.) and then solving for unknown force and displacement resultants. However, we kept finding that we had more unknowns than useful equations. This led us to realize that stiffness matrices representing objects that are not fully constrained are inherently singular, and being able to calculate meaningful forces and displacements from those matrices is only achievable with the right amount of imposed boundary conditions, i.e. constraints. Otherwise, the object can still exhibit rigid body motions.

Appendix

Geometric Parameters



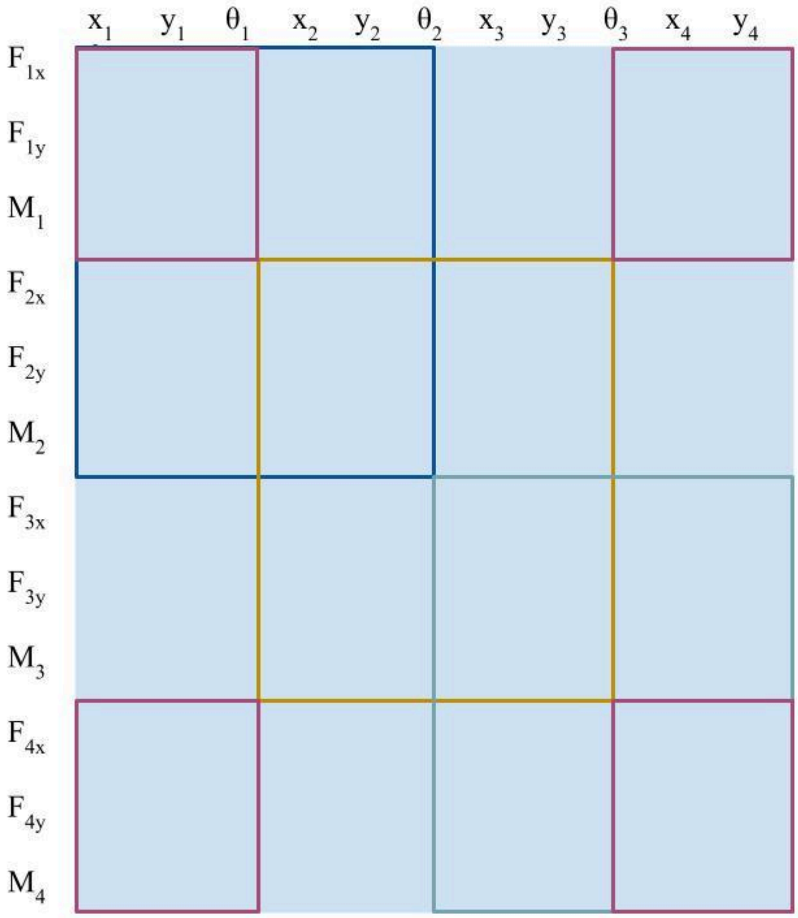
Global Stiffness Calculation Per Beam in Diamond

$$\mathbf{K} = T^{-1} * k_{beam} * T$$

$$k_{beam} = \begin{pmatrix} (A*E)/L & 0 & 0 & (-A*E)/L & 0 & 0 \\ 0 & (12*E*I)/L^3 & (6*E*I)/L^2 & 0 & (-12*E*I)/L^3 & (6*E*I)/L^2 \\ 0 & (6*E*I)/L^2 & (4*E*I)/L & 0 & (-6*E*I)/L^2 & (2*E*I)/L \\ (-A*E)/L & 0 & 0 & (A*E)/L & 0 & 0 \\ 0 & (-12*E*I)/L^3 & (-6*E*I)/L^2 & 0 & (12*E*I)/L^3 & (-6*E*I)/L^2 \\ 0 & (6*E*I)/L^2 & (2*E*I)/L & 0 & (-6*E*I)/L^2 & (4*E*I)/L \end{pmatrix}$$

$$T = \begin{pmatrix} cos(\theta) & sin(\theta) & 0 & 0 & 0 & 0 \\ -sin(\theta) & cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cos(\theta) & sin(\theta) & 0 \\ 0 & 0 & 0 & -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Global Stiffness Matrix for One Diamond



Functional Requirements

Variable	Symbol	Range	Justification	Testing (internal)
Minimum energy stored in plastic prototype per mass	E/kg	31.5 J/kg	15kg is the average mass of an ASML stage 20G is the max acceleration of an ASML stage 10 m is the max length of an ASML machine Mgh = 15kg * 20 G * 10 m = 3,000 J We decided to scale this down by 20 to get	Maximum energy storage from integrating force at max compression (8.4 cm) estimates a maximum Energy Storage. 1.37 J, however in testing we found energy stored to be higher.

			the typical crashpad energy to weight ratio for around 150 J. Average crashpad 126J/4kg = 31.5 J/kg So we chose to optimize beyond this energy to weight ratio.	We limited the height, by the bounce being resolution of the human eye M = 1.5 Kgwsx G = 9.81 Starting height = 10.4 cm Bounce Up = 0.1 mm Max compression = 5.4 cm Energy Stored = 1.5156 J Energy Stored/Mass = 40.416 J/kg
Percentage of Max Energy stored relative to energy imparted by collision	η	60%	Based on how much energy a car crash pad typically absorbs during a collision	Energy stored / initial energy = 0.51744 J /0.588 J = 88%
Stiffness ratio in y (direction of compression) (Stiffness at the beginning)/(Stiffness at max displacement)	$R_{stiffness}$	1 < $R_{stiffness}$ < 75	We chose 1 as our minimum ratio because ideally we maximize energy storage at the beginning of the collision, so at minimum we would want a linear relationship for force over the allowable range of deflection, i.e. constant stiffness. We chose 75 as our maximum ratio because we need the two stages of stiffness to be distinct in order to measure them, and our largest measurable stiffness is 75 N/mm (found in MPS 2).	$R_{stiffness} = 3.6$ Initial stiffness = 0.9 N/mm Final stiffness = 0.21 N/mm

Max distance allowed for crash (ie can't be 3 meters)	Y_{max}	< 200 mm	Maximum limited by 3D printer build plate The compression range cannot be greater than the length of the flexure.	79 mm; maximum compression
Max grid area	A	< 40,000 mm^2	Limited by 3D printer build plate (200x200mm)	14.5 x 12 cm^2 = 17,400 mm^2
Max stress in flexure grid	σ_{max}	< 5.2 MPa	3D printed TPU filament yield stress, SF 3 (15.6 MPa/3)	0.37 MPa Calculated using σ_{beam}
Mass	m	0.5 < m < 5 lbs	Can be carried easily in a person's hands. Should not weigh more than a small dumbbell.	m = 37.5 g Determined using Bambu slicer

<u>Testing Set-Up</u>